Principle of Least Action
If the initial state is $u(t_2)$ and the final state is $u(t_1)$, the optimal path between these minimizes the action integral:

$$A = \int_{t2}^{t1} \left(\text{kinetic energy} - \text{potential energy} \right) dt$$

Example:
(1) Ball of mass on attracted by gravity:

$$\begin{aligned} &U = \text{ball's hieight} \\ &K = 1/2m\left(\frac{du}{dt}\right)2 \\ &P = mgu \end{aligned}$$

$$-\frac{d}{dt}\left(m\frac{du}{dt}\right) - mg = 0$$

$$\begin{aligned} &mu''' = -mg \\ &F = mu \end{aligned}$$

(2) Pendulum with mass on

$$\begin{aligned} &K = \frac{1}{2}ml^2\left(\frac{d2}{dt}\right)^2 \\ &P = mg(l - l\cos\partial) \end{aligned}$$

$$-mgl\sin\theta - \frac{d}{dt}\left(ml^2\frac{d\partial}{dt} = 0\right)$$

$$\frac{d^2\partial}{dt^2} + \frac{y}{l}\sin\partial = 0$$

<ins>2D Problems</ins>

$$P(u) = \iint_S \left[\frac{c}{2}\left(\frac{2u}{2x}\right)^2 + \frac{c}{2}\left(\frac{2u}{2y}\right)^2_{dxdy} - f(x,y)u(x,y)\right]$$

$$-\frac{d}{dx}\left(c\frac{2u}{2x}\right) - \frac{2}{2y}\left(c\frac{2u}{2y}\right) = f(x,y)$$

In general,

$$P\iint_s a\left(\frac{2u}{2x}\right)^2 + 2b\left(\frac{2u}{2x}\right)\left(\frac{2u}{2y}\right)$$

$$+\subset\left(\frac{2u}{2y}\right)^2 \quad dxdy$$

$$\Rightarrow \quad au_{xx} + 2bu_{xy} + cu_{yy} = 0$$

Quadratic form

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

(i)

$ac\rangle b^2 \quad : \quad elliptic$

$a = c = 1, b = 0$

$u_{xx} + u_{yy} = 0$

(ii)

$ac\langle b^2$

$a = 1, c = -1, b = 0 \; u_{xx} = u_{yy}, \quad h, petholic$

(iii)

$ac = b2$

$a = 1$

$c = 0$

$b = 0$

$u_t = u_{xx}$

Snakes

$$\underset{\vee}{\rightarrow} : [0, \quad 1] \rightarrow \Re^2$$

$$\underset{\vee}{\rightarrow}(0) = \underset{\vee}{\rightarrow}(1)$$

$$\sum\left(\underset{\vee}{\rightarrow}\right) = \sum_s \underset{\vee}{\rightarrow} + P\left(\underset{\vee}{\rightarrow}\right)$$

$$\sum_s\left(\underset{\vee}{\rightarrow}\right) = \int_s^1\left(w_1(s)\left|\underset{s}{\underset{\vee}{\rightarrow}}\right|^2 + w_2(s)\left|\underset{ss}{\underset{\vee}{\rightarrow}}\right|^2\right)ds$$

Dynamics: Constant dynamical system and allows it I ensure at a minimal every state so it achieves equilibrium. Use Lagragion dynamics is Principle of heart action.

$$\vec{\vee}(s,t)$$

$$\frac{1}{2}\int_0^1 u\left|\vec{v}_t\right|^2 ds$$

$$\zeta\left(\vec{\vee}\right)=\frac{1}{2}\underbrace{\int}_{\substack{kinetic\\ eneergy}} u\left|\vec{v}_t\right|^2 ds-\frac{1}{2}\sum\left(\vec{v}\right)$$

$$v(s_v\ t_0)=inital$$
$$v(s_v\ t_1)=\ final$$
$$S_v\int_{t_0}^{t_1}\Im\left(\vec{v}\right)dt=0$$

$$S_v\left(\frac{1}{2}\int_{t_0}^{t_1}\int_0^1 u\left|\vec{v}_t\right|^2-w_1(s)\left|\vec{v}_s\right|^2-w_2(s)\left|\vec{v}_{ss}\right|^2-P\left(\vec{v}\right)ds\quad dt\right)=0$$

Need to dissipate S_v energy

Raleigh dissipation functional:

$$D(x_t)=\frac{1}{2}\int_0^1 \vartheta\left|v_t\right|^2\ ds\to\vartheta(s)$$

$$\frac{d}{dt}\left(\frac{2L}{2\,\vec{v}_t}\right)+\frac{2D}{2\,\vec{v}_t}-\frac{2L}{2\,\vec{v}}+\frac{2}{2s}\left(\frac{2L}{2\,\vec{v}_s}\right)-\frac{2}{2s^2}\left(\frac{2L}{2\,\vec{v}_{ss}}\right)=0$$

Assume $u(s)=u,\quad \gamma(s)=\gamma$

$$u\vec{v}_{tt}+\gamma\,\vec{\mathrm{v}}_{\mathrm{t}}-\frac{2}{2s}\left(w,\vec{v}_s\right)+\frac{2^2}{2s^2}\left(w_2\ \vec{v}_{ss}\right)=-\underbrace{\nabla P\left(\vec{v}(s,t)\right)}_{\text{couples arche to the image data}}$$